EXTENSION OF ASYMPTOTIC RELATIVE LAWS OF FRICTION AND HEAT TRANSFER TO NONISOTHERMAL FLOW OF A GAS AT FINITE REYNOLDS NUMBERS

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It is shown that asymptotic relative laws of friction and heat transfer obtained theoretically are also valid for finite Reynolds numbers R^{**} if "standard" local values of the friction coefficient C_{f_0} and the Stanton number S_0 are chosen in a certain way.

The value of the quantity $\Psi_1 = (C_f/C_{f01})$ for a nonisothermal boundary layer is found theoretically in monograph [1]. Here C_f is the local value of the friction coefficient under given conditions, $C_{f_{01}}$ is for isothermal friction, zero transverse mass flux and no longitudinal pressure gradient. In this case $C_{f_{01}}$ is computed by means of known relationships for an incompressible fluid.

$$C_{f01} = f(R_1^{**}), \qquad R_1^{**} = \frac{w_0\rho_0\delta^{**}}{\mu_0}. \tag{1}$$

Here R_1^{**} is the Reynolds number corresponding to the conditions considered; δ^{**} is the momentum thickness; and w_0, ρ_0, μ_0 are respectively the velocity, density, and dynamic viscosity at the outer edge of the boundary layer; ρ_0 and μ_0 are referred to the temperature of the external flow T_0. The initial equation for calculating Ψ_1 is [1]

$$\Psi_1 = \left(\frac{1}{Z}\int\limits_{\omega_1}^{1}\left(\frac{\rho\tau_0}{\rho_0\tau}\right)^{1/2} d\omega\right)^2,$$

$$Z = \left(\frac{C_{f01}}{2}\right)^{1/2}\int\limits_{y_1}^{\infty}\left(\frac{\tau_0}{1-\beta}\right)^{1/2}\frac{dy}{l}. \tag{2}$$

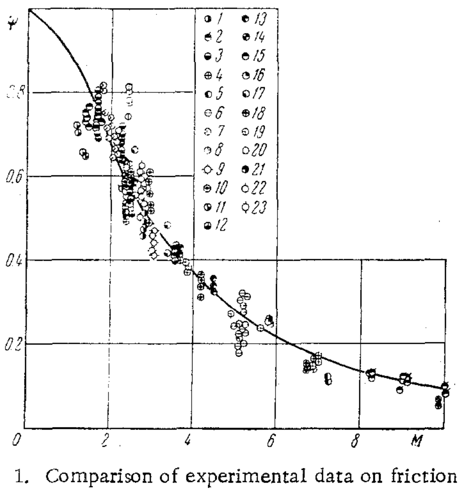

Fig. 1. Comparison of experimental data on friction and heat transfer with theoretical values. The curve represents the asymptotic relationship (5); the points: 1) Kozlov [5], 2) Hill [8, 9], 3) Papas [10], 4) Sommer and Short [11], 5) Abbot [3], 6) Winkler [12], 7) Wilson [13], 8) Rubesin and others [14], 9) Brinich and Diaconis [15], 10) Cope [16], 11) Dahwan [17], 12) Chapman and Kester [18], 13) Coles [19], 14) O'Donnell [20], 15) Hakkinen [21], 16) Korkegi [22], 17) Bradfield and deCoursin [23], 18) Matting and others [24], 19) Lobb and others [25], 20) Monahan and others [3], 21) Monahan and Cook [3], 22) Spivak [3], 23) Monahan and Johnson [3].

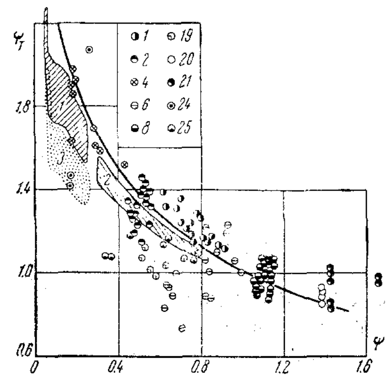

Fig. 2. Comparison of experimental data on friction and heat transfer with theoretical values. The curve represents the asymptotic relationship (7); friction data—points: 1) Kozlov [5], 2) Hill [8, 9], 4) Sommer and Short [11], 6) Winkler [12], 24) Abbot [3], 20) Monahan, 21) Monahan and Cook, 19) Lobb and others [25]; region 1—Belyanin [6]; heat transfer data—points: 3) Papas [10], 25) Mukhin [7]; regions: 2) Komarov [4], 3) Belyanin [6].

Here y_1, ω_1 are the thickness of the viscous sublayer and the dimensionless velocity on its boundary, respectively; ρ is the density in the boundary layer; τ, τ_0 are the relative laws of variation of the shear stresses over the thickness of the boundary layer for the given conditions and for isothermal conditions, respectively, for zero transverse mass flux and no longitudinal pressure gradient; β is a coefficient which takes account of the effect of density fluctuations on momentum transfer; and l is the mixing length. As shown in [2, 3], calculations based on (2), taking account of the finite Reynolds numbers R_1^{**}, after the introduction of Z and ω_1 give to the first approximation a standard deviation from the available experimental data of about 12%.

In deriving equation (2) no restrictions of any sort were imposed on the choice of "standard" conditions. Let us introduce a new quantity defined as

$$C_{f_0} = f(R^{**}), \qquad R^{**} = \frac{w_0\rho_0\delta^{**}}{\mu_w}. \tag{3}$$

Here μ_w is the dynamic viscosity found from the wall temperature T_w.

Substituting (3) into (2) and passing to the limit $R^{**} \to \infty$, we find, as in [1], that $Z \to 1$, $y_1 \to 0$, $\omega_1 \to 0$, and

$$\int\limits_0^1\left(\Psi\frac{\rho_0\tau}{\rho\tau_0}\right)^{-1/2} d\omega = 1. \tag{4}$$

Since this expression was derived for flows with vanishing viscosity, it naturally should not depend on the method of determining viscosity. Correspondingly, the form of the equations for Ψ, which were derived in [1], also does not change. For a nonisothermal boundary layer on an impermeable plate with $R^{**} \to \infty$, Ψ is given by the expression

$$\Psi = \frac{C_f}{C_{f_0}} = \left[\frac{2}{\sqrt{\psi}+1}\,\frac{\operatorname{arc\,tg} M\sqrt{{}^1\!/_2 r(k-1)}}{M\sqrt{{}^1\!/_2 r(k-1)}}\right]^2,$$

$$\psi = \frac{T_w}{T_w^*} \qquad \left(T_w^* = T_0\left(1 + r\frac{k-1}{2}M^2\right)\right). \tag{5}$$

The experimental data of different authors [3-25] for adiabatic conditions and heat transfer with an asymptotic law [5] are compared in Fig. 1.

The values of C_{f_0} were computed from the formula

$$C_{f_0} = 0.0252\,(R^{**})^{-0.25}\,. \tag{6}$$

For the case of heat transfer, the experimental points are reduced to thermally-insulated conditions and the following quantity is plotted along the axis of ordinates:

$$\frac{C_f}{C_{f_0}}\left(\frac{\psi^{0.5}+1}{2}\right)^2.$$

As can be seen from the graph, if C_{f_0} is determined from formula (3), the agreement of the asymptotic relationship with the experimental data is quite good.

In Fig. 2, experimental data are compared with data from the asymptotic relationship

$$\Psi_w = \left(\frac{2}{\psi^{0.5}+1}\right)^2, \tag{7}$$

which takes the effect of nonisothermicity due to heat transfer into consideration. The experimental results obtained with supersonic flows were reduced to conditions of incompressibility by means of the formula

$$\frac{C_f}{C_{f_0}}\left(\frac{M\sqrt{{}^1\!/_2 r\,(k-1)}}{\operatorname{arc\,tg} M\sqrt{{}^1\!/_2 r\,(k-1)}}\right)^2 = f(\psi).$$

These graphs show that the asymptotic relationship agrees satisfactorily with experimental data for these conditions, too. The experiments of [7, 12, 25] are exceptions.

Thus, going over to a new standard C_{f_0} in the relative quantity Ψ justifies us in using the limit values of Ψ and neglecting the effect of finite R^{**} numbers in calculating a nonisothermal turbulent boundary layer. As implied by Figs. 1 and 2, similar conclusions can also be drawn for the law of heat transfer. In this case, the Stanton number

$$S_0 = f(R_T,\ P)\left(R_T{}^{**} = \frac{w_0\rho_0\delta_T}{\mu_w}\right).$$

Here δ_T^{**} is the energy thickness and P is the Prandtl number determined by the flow temperature. When processing the data of references [7, 10, 23] and of V. P. Komarov [4], which were obtained at values of the R_T^{**} number ranging from 1000 to 30 000, the Stanton number S_0 was found from the formula

$$S_0 = 0.0126 R_T{}^{**-0.25} P^{-0.75}. \tag{8}$$

The experiments on heat transfer and friction [6] were conducted at comparatively low Reynolds numbers R_T^{**} (200-500). For this range of R_T^{**} numbers, the values of S_0 were computed from the relationship $S_0 = (1/2)C_{f_0}P^{-0.75}$ and C_{f_0} from the formula recommended in [3].

The result obtained on the applicability of asymptotic formulas is not unexpected if we bear the following facts in mind. When $R^{**} \to \infty$ the quantity Ψ determined from [4] takes account of the effect of nonisothermicity only through changes in the density in the boundary layer. The variable viscosity in the laminar sublayer can affect the relative laws of friction and heat transfer only at finite R^{**} numbers. In this case it is possible to assume that the quantity Ψ remains as when $R^{**} \to \infty$, since the effect of variable density in the laminar sublayer, where viscous friction predominates, cannot be important. In this case it is wise to choose "standard" values of C_{f_0} (or S_0) that take into account only those nonisothermicity, effects that are manifested in the laminar sublayer, that is, take variable viscosity into account. We may assume that expression (3) satisfies these requirements. Experiments with nonisothermal flows of liquids in droplet form, when the density in the boundary layer is constant and the viscosity variable, afford some confirmation of this.

Under these conditions the effect of nonisothermicity on the law of friction can be excluded, in the first approximation, if the dynamic viscosity coefficients in the R^{**} number are determined from the wall temperature [26-29].

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